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A new semi-active suspension control strategy through LPV technique

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ABSTRACT

This paper is concerned with the design and the analysis of a semi-active suspension controller. In the recent years different kinds of semi-active control strategies, like two-state Skyhook, LQ-clipped or model-predictive control, have already been developed in the literature. In this paper, a new semi-active suspension control strategy that a priori satisfies the principal limitations of a semi-active suspension actuator (dissipative constraint and force bounds) is introduced using the tools of the linear parameter varying (LPV) theory. This new approach exhibits some interesting advantages (implementation, performance flexibility, robustness, etc.) compared to already existing methods. Both industrial criterion based evaluation and simulations on a nonlinear quarter vehicle model are performed to show the efficiency of the method and to validate the theoretical approach.

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1. Introduction

Global vehicle control is now an important issue in the future of vehicle control technology (see Shibahata, 2005). Many systems are involved in cars to guarantee both safety (e.g. ABS, ESP, etc.) (see, e.g., Corno, Savaresi, Tanelli, & Fabbri, 2008) and comfort (e.g. suspensions, seats, etc.). Concerning vehicle comfort and road holding in normal cruise situation (i.e. not in emergency case like over-steering or rolling situations), it is well admitted that controlled suspension systems can provide good passenger isolation from road unevenness while keeping admissible the road holding performances. Industrial and academic researches are very active in the automotive field; suspension design and control are important aspects for comfort and safety achievements (see Fischer & Isermann, 2003; Giorgeou, Verros, & Natsiavas, 2007; Hrovat, 1997; Ieluzzi, Turco, & Montiglio, 2005; Kawabe, Isobe, Watanabe, Hanba, & Miyasato, 1998).

In this area, an extensive literature and many models are available and widely used. In the last decade, many different active suspension system control approaches were developed. The Skyhook control (see Poussot-Vassal, Sename, Dugard, Ramirez-Mendoza, & Flores, 2006; Sammier, Sename, & Dugard, 2003), suits well to improve comfort but is limited to improve road holding. The linear quadratic control (see Hrovat, 1997) can provide both comfort and road holding improvements but requires the full state measurement or estimate. Linear time invariant (LTI) $\mathscr{H}_\infty$ control (see Rossi & Lucente, 2004; Sammier et al., 2003; Zin, Sename, & Dugard, 2005) can achieve better results improving both comfort and road holding ensuring a defined frequency behavior but is limited to fixed performances (due to fixed weights). Mixed LTI $\mathscr{H}_\infty/\mathscr{H}_2$ approaches (see Abdellahi, Mehdi, & Saad, 2000; Gáspár, Szaszi, & Bokor, 1998; Lu, 2004; Lu & DePoyster, 2002; Poussot-Vassal et al., 2006; Takahashi, Camino, Zampieri, & Peres, 1998; Tuan, Apkarian, & Hosoe, 2001) that can improve $\mathscr{H}_\infty$ control reducing signals energy. Recently, the use of linear parameter varying (LPV) approaches (see Fialho & Balas, 2002; Gáspár, Szaszi, & Bokor, 2004; Poussot-Vassal, Drivet, Sename, Dugard, & Ramirez-Mendoza, 2007) leads to controllers that can either adapt the performances according to measured signals (road, deflection, etc.) or improve robustness, taking account of the nonlinearities (see Poussot-Vassal et al., 2006; Zin, 2005; Zin, Sename, Gáspár, Dugard, & Bokor, 2006, 2008).

Most of these controllers are designed and validated assuming that the actuator of the suspension is fully active. Unfortunately, such active actuators are not yet used on a wide range of vehicles because of their inherent cost (e.g. energy, weight, volume, price, etc.) and low performance (e.g. time response); hence, in the industry, semi-active actuators (e.g. controlled dampers) are often preferred. Even if efforts on bounding the actuator force have been done using time domain (e.g. Chen & Guo, 2005) or frequency domain (e.g. Gáspár et al., 2004) constraint, the controller does not fulfill the dissipative constraint of the actuator. Hence, the performances obtained in simulation will be lost after implementation due to the controlled damper structural limitations. Usually, the method is to build an active controller and then, make it semi-active by saturating the forces required (known as

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the "clipped" approach), which does not ensure internal stability and performances any longer.

In the last years, different kinds of strategies have been developed to tackle such limitations, but semi-active suspension control remains an open research area. The two-state Skyhook control is an on/off strategy that switches between high and low damping coefficients in order to achieve body comfort specifications. Mixed Skyhook and ADD (see Savaresi, Silani, & Bittanti, 2004; Savaresi & Spelta, 2007) is also a comfort oriented control strategy involving switching strategy. Clipped approaches lead to unpredictable behaviors and reduce the achievable performances (see Canale, Milanese, & Novara, 2006; Giorgetti, Bemporad, Tseng, & Hrovat, 2006). In Giorgetti et al. (2006), authors compare different semi-active strategies based on optimal control and introduce a hybrid model-predictive optimal controller. The resulting control law is implemented via a hybrid controller that switches between a large number of controllers (function of the prediction horizon) and requires a full state measurement. In Canale et al. (2006), another model-predictive (MPC) semi-active suspension is proposed and results in good performances compared to the Skyhook and LQ-clipped approaches but it requires an on-line "fast" optimization procedure. As it involves optimal control, full state measurement and a good knowledge of the model parameters are necessary (see also Giua, Melas, Seatzu, & Usai, 2004).

The contribution of this paper is to propose new results to design a semi-active suspension controller. The methodology is developed for the first time through the LPV theory which allows to ensure internal stability and some performance criteria for the semi-active suspension. The main interest of such an approach is that it a priori allows to fulfill the dissipative actuator constraint and enable the designer to build a controller in the robust framework ($\mathscr{H}_\infty$, $\mathscr{H}_2$, mixed, etc.) which is appreciated in the industry because of its flexibility (due to the weight functions) and its robustness to modeling uncertainties. As long as the new methodology does not involve any on-line optimization process and only requires a single sensor, it could be an interesting algorithm from the application's point of view. The proposed methodology is illustrated on a semi-active suspension equipped with a magneto-rheological (M-R) damper. The authors stress that the proposed design can be applied to any semi-active suspension actuators (not restricted to M-R dampers).

The paper is organized as follows. Section 2 introduces the nonlinear quarter car models used for synthesis, performance evaluation and simulation. In Section 3, the involved semi-active suspension actuator system (based on real experimental data) is described. In Section 4, a quarter vehicle frequency based industrial criterion is introduced in order to tune and evaluate the control strategy. Section 5 is devoted to recall the background and some facts on the LPV theory. In Section 6, the proposed semi-active LPV/$\mathscr{H}_\infty$ control design and its scheduling strategy are presented. In Section 7, both performance evaluation using the introduced criterion and time domain simulations are done on a nonlinear quarter vehicle model to show the efficiency of the proposed method. Conclusions on this new semi-active suspension approach and perspectives are discussed in Section 8.

2. Quarter car model

In the sequel, the involved passive reference and controlled nonlinear quarter vehicle models are described. Such a quarter car model is suitable when suspension control has to be synthesized. It makes possible to analyze the vertical behavior.

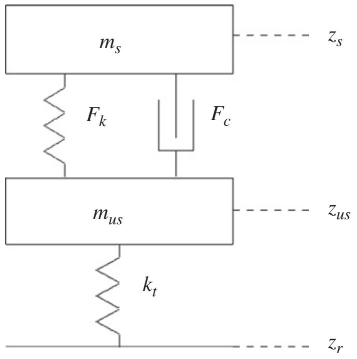


Fig. 1. Passive suspension model.

2.1. Passive nonlinear suspension model

The simplified quarter vehicle model involved here includes the sprung mass (m_s) and the unsprung mass (m_{us}) and only catches vertical motions (z_s, z_{us}). As its damping coefficient is negligible, the tire is simply modeled by a spring linked to the road (z_r) where a contact point is assumed. The passive suspension, located between m_s and m_{us}, is modeled by a damper and a spring (Fig. 1).

The nonlinear passive model, that will be later used as a reference model (for performance evaluation and comparison with the controlled one), is given by

$$\begin{cases} m_s\ddot{z}_s = -F_k(z_{def}) - F_c(\dot{z}_{def}) \\ m_{us}\ddot{z}_{us} = F_k(z_{def}) + F_c(\dot{z}_{def}) - k_t(z_{us} - z_r) \\ z_{def} \in [\underline{z}_{def}\ \ \overline{z}_{def}] \end{cases} \tag{1}$$

where $F_k(z_{def})$ and $F_c(\dot{z}_{def})$ (with $z_{def} = z_s - z_{us}$ and $\dot{z}_{def} = \dot{z}_s - \dot{z}_{us}$) are the nonlinear forces provided by the spring and damper, respectively (Fig. 2).

The involved model parameters have been identified on a "Renault Mégane Coupé" car and are given in Table 1 (see Zin, Sename, Basset, Dugard, & Gissinger, 2004).

2.2. Controlled suspension

In the controlled suspension framework, one considers the model given in Fig. 3.

Note that in this formulation, the passive damper (F_c) appearing in Eq. (1) is replaced by an actuator or a controlled damper (u), which is consistent with the reality. Then the controlled quarter car model can be derived as follows:

$$\begin{cases} m_s\ddot{z}_s = -F_k(z_{def}) + u \\ m_{us}\ddot{z}_{us} = F_k(z_{def}) - u - k_t(z_{us} - z_r) \\ z_{def} \in [\underline{z}_{def}\ \ \overline{z}_{def}] \end{cases} \tag{2}$$

where u is the control input of the system, provided by the considered suspension actuator.

3. Semi-active suspension actuator

In the previous section, the control input u was introduced to control the quarter car model. Since here the focus is done on semi-active suspension control, in the sequel, emphasis is put on static performances and structural limitations for the considered semi-active actuator.

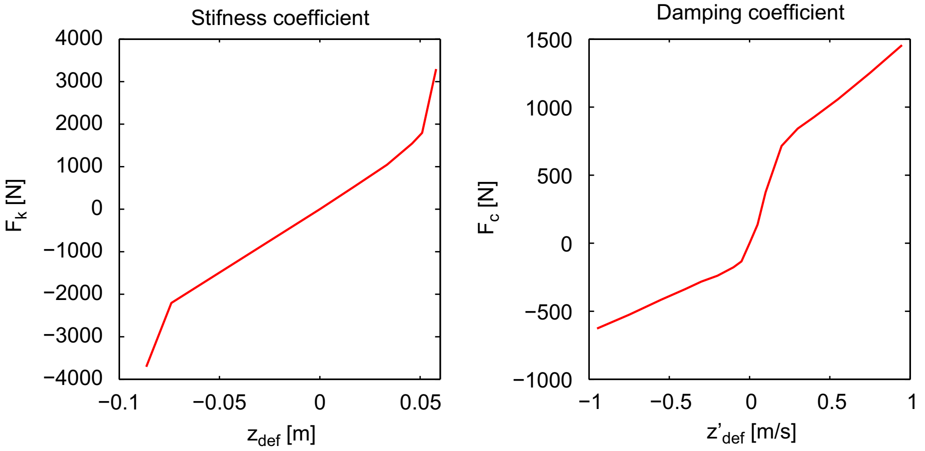


Fig. 2. Nonlinear forces provided by the spring (left) and the damper (right).

Table 1
Parameters model of a “Renault Mégane Coupé”

Symbol	Value	Description
m_s	315 kg	Sprung mass
m_{us}	37.5 kg	Unsprung mass
k	29 500 N/m	Suspension linearized stiffness
c	1500 N/m/s	Suspension linearized damping
k_p	210 000 N/m	Tire stiffness
$[\underline{z}_{def}, \overline{z}_{def}]$	$[-8, 6]$ cm	Suspension's deflection limits

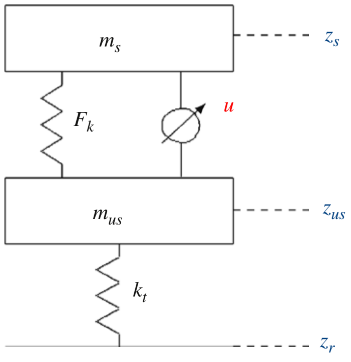


Fig. 3. Controlled suspension model.

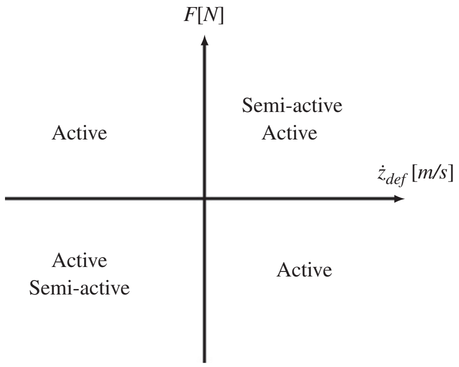


Fig. 4. Active vs. semi-active force/deflection speed space.

3.1. *Active vs. semi-active suspension systems*

Active suspension systems can either store, dissipate or generate energy to the masses (m_s and m_{us}). When semi-active suspension actuators are considered, only energy dissipation is possible. This fundamental difference is usually represented using the force/deflection speed space representation given in Fig. 4. Hence, a semi-active controller can only deliver forces within the two semi-active quadrants. Note that when a full active actuator is considered, the delivered force can belong to all the four quadrants.

3.2. *A semi-active actuator: the M-R damper*

In the automotive industry, semi-active actuators, like controlled dampers, are being more and more used because they can provide a varying damping coefficient to the suspension system while being of the same weight as passive dampers (where only one damping coefficient is set). For that purpose automotive manufacturers, like Lord, Delphi, Sachs (see Delphi, 2005, 2007; Lord, 2007; Sachs, 2007), sell different kinds of such actuators: either pneumatic, hydraulic, magneto-rheological, etc. Even if the proposed control strategy can be applied to any kind of semi-active dampers, in the proposed application, the magneto-rheological case will be considered (increasingly studied and used in the industry because of its great performances) (see Du, Yim Sze, & Lam, 2005; Gravatt, 2003; Koo, Goncalves, & Ahmadian, 2004). The actuators considered here are semi-active continuously controlled dampers (denoted as CCD). For this type of controlled damper, it is assumed that all the forces within the allowed semi-active space quadrants can be achieved (Fig. 4). The working principle of the M-R damper is the following: the viscosity of the rheological fluid inside the damper (i.e. the damping coefficient) can be changed by adjusting the current value (I_{MRD}). The main advantage of such an actuator is that the weight and the volume are similar to classic passive dampers and the range of damping coefficients is nearly infinite; one can “choose” all the damping coefficients needed within the allowable space. In the meantime, the time response is very fast (about 10 ms), compared to an active hydrological actuator (see Koo et al., 2004).

For this purpose, a Delphi M-R damper is considered. It is available at the Tecnologico de Monterrey (see Delphi, 2005; Nino, Ramirez-Mendoza, & Guerre, 2006) and is given in Fig. 5. To evaluate the upper and lower limitations of this actuator, a sinusoidal disturbance of frequency 0.25 Hz is generated at the extremity of the suspension (equivalent to a deflection disturbance) for different magnitudes of current in order to measure the achievable forces of this damper. Fig. 6 shows results for two different current values. For a better accuracy, an extensive study of the nonlinear, hysteresis and dynamical effects should be done considering different current and frequency experimentations. As for control purpose only the static model of the considered semi-active actuator is required, here it is simply modeled as static upper and lower bounds (as shown in Fig. 7).

3.3. *Semi-active suspension static model*

Based on the force/deflection speed diagram given in Fig. 6, a static model of the considered damper is derived. It simply consists of upper and lower bounds of the achievable forces of the damper and will be denoted as a function $D(\dot{z}_{def})$. Usually (as in the "clipped" approach), for a given deflection speed ($\dot{z}_{def}$), if the controller computes a force F^* out of the achievable damper range, the force provided to the system will be the projection $F^{\perp}$ of F^* on the admissible force area (see Fig. 7).

Fig. 5. Delphi M-R damper.

This projection, usually used to ensure the semi-activeness, introduces limitations in the suspension performances. Here the proposed method tends to avoid such a projection.

Remark. In this work, an M-R damper is involved, because real experimental data have been provided by the TEC of Monterrey, in a collaborative project (Drivet et al., 2006; Flores et al., 2006; Nino et al., 2006; Poussot-Vassal et al., 2006). While this kind of damper is well studied for its interesting properties, it could be replaced by any kind of semi-active damper. The proposed approach will only require upper and lower bounds of the actuator.

4. Performance objectives and evaluation

In this section both nonlinear frequency computation and performance evaluation are described. Controller tuning and

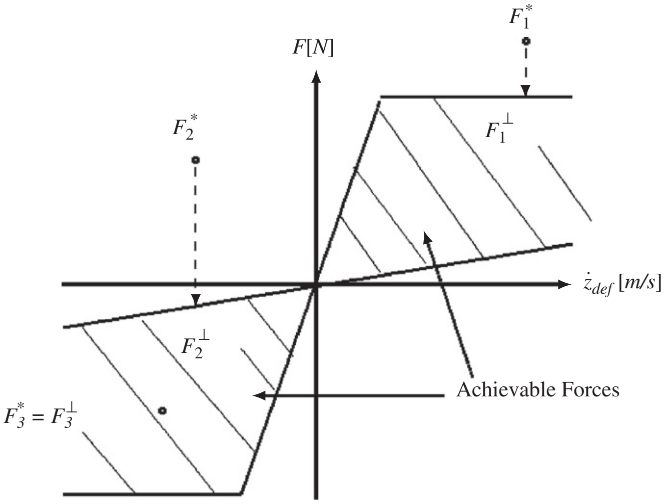


Fig. 7. Illustration of projection principle of the semi-active controlled damper model (F_1^* and F_2^* are out of the allowed area and F_3^* is inside).

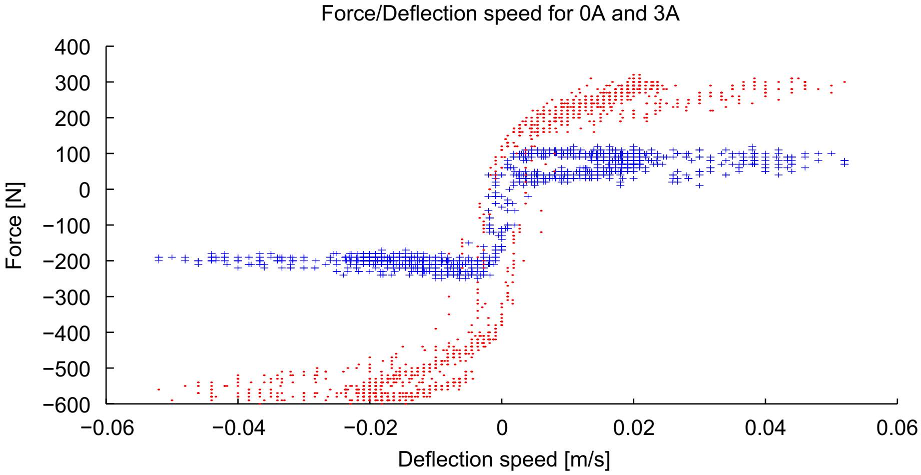


Fig. 6. Experimental force/deflection speed diagrams of the Delphi suspension for different current inputs (0A cross and 3A dots).

closed-loop evaluation are done (in Section 6) according to this frequency based performance measure.

4.1. "Frequency" response of the passive nonlinear suspension model

The passive nonlinear model of the "Renault Mégane Coupé" described by Eq. (1) will be used as a reference model w.r.t. the controlled nonlinear one described in (2) with the corresponding control law. As they are both nonlinear, the Bode diagram representations cannot be computed anymore. Here the frequency plots of the nonlinear model are introduced (function of the input magnitude and frequency). These plots are obtained with a sinusoidal input z_r of varying magnitude (between 1 and 8 cm) over 10 periods with varying frequencies (between 0.1 and 30 Hz). Note that studying the frequency behavior under this magnitude range makes possible to enter into the nonlinearities of the suspension. Then a discrete Fourier Transform is performed to evaluate the system gain. Fig. 8 shows the frequency responses of $\ddot{z}_s/z_r$, z_s/z_r, z_{us}/z_r and z_{def}/z_r for the nonlinear passive quarter vehicle model; note that the results are given using frequency logspace and magnitude converted into decibels in order to remind the well known Bode representation.

This representation as a function of both frequency and magnitude exhibits the important influence of the suspension nonlinearities on the quarter car behavior. This will be taken into account in the performance evaluation of the proposed strategy.

4.2. Frequency based industrial performance criterion

In order to estimate the comfort, the vertical motion (z_s) and acceleration ($\ddot{z}_s$) of the chassis have to be studied. The wheel vertical motion (z_{us}) and the suspension deflection (z_{def}) are related to road holding specifications (see Sammier, 2001; Zin, 2005). In the following, four performance objectives are derived from industrial control specifications (Sammier et al., 2003) that are consistent with the one given in Gillespie (1992) book:

(1) *Comfort at high frequencies*: The vibration isolation between 4 and 30 Hz is evaluated by the transfer function $\ddot{z}_s/z_r$. The vertical acceleration of the chassis has to be limited in order to obtain good comfort at high frequencies (>5 Hz), although the human body is not sensitive to vertical accelerations at high frequencies (>10 Hz).
(2) *Comfort at low frequencies*: The vibration isolation between 0 and 5 Hz is evaluated by the transfer function z_s/z_r. Ideally, the vertical displacement of the chassis should be the same as that of the road for low frequencies (lower than around 1 Hz) and null for high frequencies (higher than around 1 Hz). In practice, for low disturbances ($z_r<3$ cm), the maximal gain occurring between 1 and 5 Hz of z_s/z_r has to be bounded by 1.8.
(3) *Road holding*: As indicated before, it is evaluated with the transfer function of z_{us}/z_r. For a good road holding, the maximal gain, in the range 0–20 Hz, of the considered transfer function has to be limited to 1.8 (for low disturbance).
(4) *Suspension constraints*: The transfer z_{def}/z_r is a road holding indicator and also a constraint on the deflection of the actuator evaluated between 0 and 20 Hz in order to preserve its life cycle.

In each case the issue is to perform better than a passive suspension does. Therefore, to compare the control approach proposed thereafter with the passive one, the power spectral density (PSD) measure of each of these signals along the frequency and magnitude space of interest is used as the following formula:

$$I_{\{f_1,a_1\}\to\{f_2,a_2\}}(x)=\sqrt{\int_{f_1}^{f_2}\int_{a_1}^{a_2} x^2(f,a)\,\mathrm{d}a\cdot\mathrm{d}f} \qquad (3)$$

where f_1 and f_2 (resp. a_1, a_2) are the lower and higher frequency (resp. magnitude) bounds, respectively, and x is the signal of interest.

Remark. The criteria used in this paper have been derived according to industrial criteria. For further information on these criteria, the reader is invited to read Sammier et al.'s (2003) paper, which is a previous work from two of the authors.

5. Background on LPV systems and gain-scheduled $\mathscr{H}_\infty$ polytopic controller

Before introducing the main result of this paper, which uses well known LPV and robust results, let us first recall some basic facts on LMI based $\mathscr{H}_\infty$ problem resolution for LPV systems.

Thanks to recent works on robust linear control (see Apkarian & Gahinet, 1995; Chilali & Gahinet, 1996; Chilali, Gahinet, & Apkarian, 1999; de Souza & Trofino, 2005; Gahinet, Apkarian, & Chilali, 1996; Scherer, 2004; Scherer, Gahinet, & Chilali, 1997) and references therein, LMI and optimization tools (see, e.g. Benson, Ye, & Zhang, 2000; Boyd, El-Ghaoui, Feron, & Balakrishnan, 1994; Lofberg, 2004; Scorletti, 2004; Sturm, 1999) and references therein, robust controller synthesis theory is well established and widely used in control applications. This approach makes possible to tackle nonlinearities of a system by considering them as parameter uncertainties and to build robust controllers w.r.t. these uncertainties. However, as long as it only recovers linear systems, robust approaches lead to very conservative results when either high performances are looked for or when the system nonlinearities are large. LPV theory allows to model the nonlinearities or to make the controller performances varying through the linear introduction of parameters. Hence, since a decade ago, LPV model is increasingly being used and allows to extend classical linear robust control methodology to a larger class of systems, keeping the use of linear tools. Many studies, either theoretical or practical, have been developed in the last few years (see Bruzelius, 2004; Bruzelius, Pettersson, & Breitholtz, 2004; Fialho & Balas, 2002; Gáspár, Szaszi, & Bokor, 2003, 2004; Pellandra, Apkarian, Tuan, & Alazard, 2004; Rahar-ijaona, 2004; Zin, 2005; Zin et al., 2006). Here some basic definitions of an LPV system, controller and closed-loop are recalled.

Definition 1. LPV generalized system.

A dynamical LPV system can be described in the following form:

$$\Sigma(\theta):\begin{bmatrix}\dot{x}\\ z\\ y\end{bmatrix}=\left[\begin{array}{c|cc}A(\theta) & B_1(\theta) & B_2(\theta)\\ \hline C_1(\theta) & D_{11}(\theta) & D_{12}(\theta)\\ C_2(\theta) & D_{21}(\theta) & D_{22}(\theta)\end{array}\right]\begin{bmatrix}x\\ w\\ u\end{bmatrix} \qquad (4)$$

where x, w and u define the state, the exogenous and control input, respectively; z and y hold for the controlled output and system measure, respectively. $\theta(.)\in\Theta$ is the set of varying parameters that describe a set of systems. $A\in R^{n\times n}$, $B_1\in R^{n\times n_w}$, $B_2\in R^{n\times n_u}$, $C_1\in R^{n_z\times n}$, $D_{11}\in R^{n_z\times n_w}$ and $D_{12}\in R^{n_z\times n_u}$, $C_2\in R^{n_y\times n}$, $D_{21}\in R^{n_y\times n_w}$ and $D_{22}\in R^{n_y\times n_u}$.

Definition 2. LPV controller.

An LPV controller is defined by

$$S(\theta):\begin{bmatrix}\dot{x}_c\\ u\end{bmatrix}=\begin{bmatrix}A_c(\theta) & B_c(\theta)\\ C_c(\theta) & D_c(\theta)\end{bmatrix}\begin{bmatrix}x_c\\ y\end{bmatrix} \qquad (5)$$

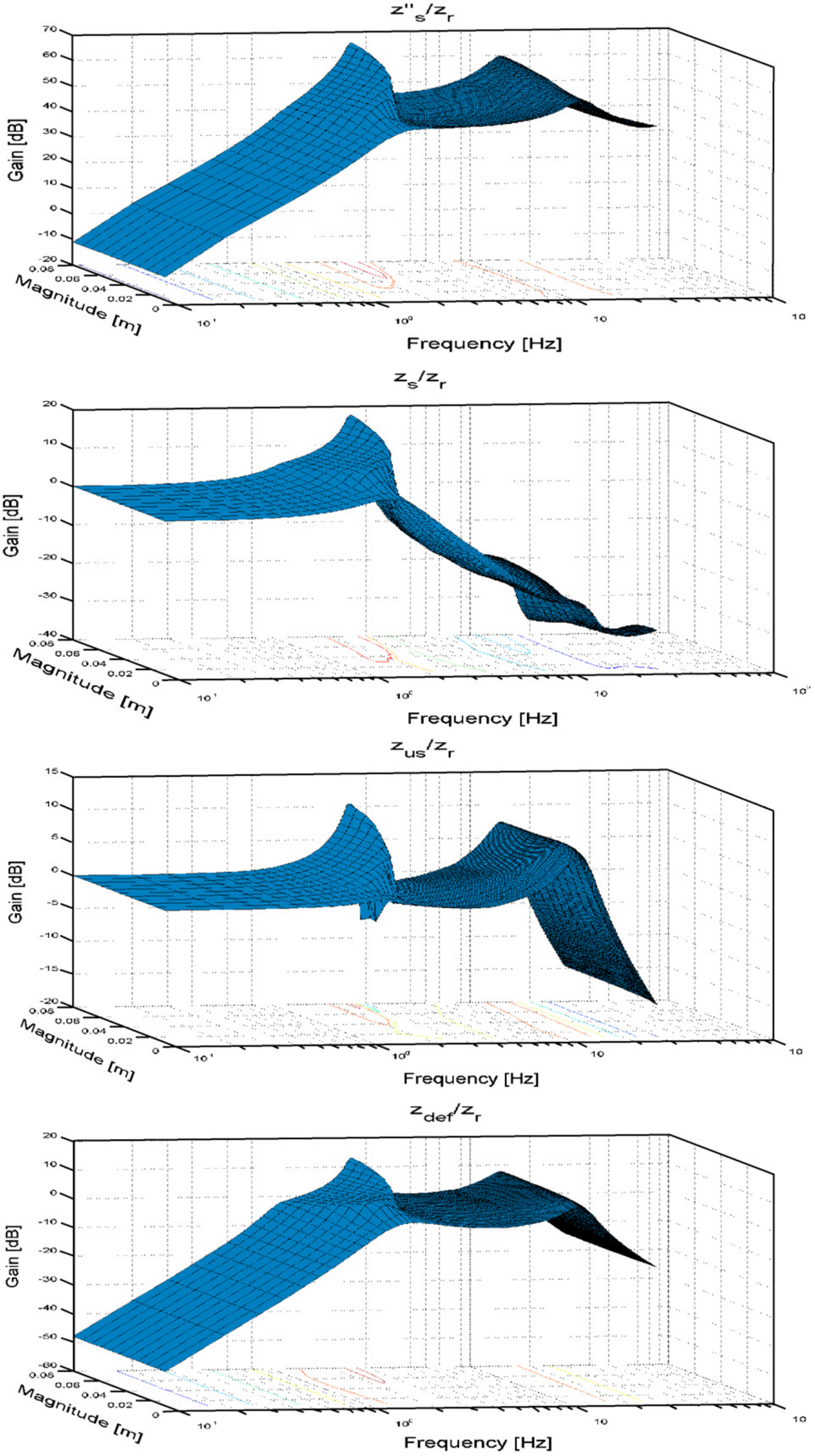


Fig. 8. Frequency responses of the quarter vehicle.

where x_c, y and u are the state, the control input and output, respectively, of the controller associated to the system (4). $\theta(.) \in \Theta$ is the set of the varying parameters associated to the controller. $A_c \in R^{n \times n}$, $B_c \in R^{n \times n_y}$, $C_c \in R^{n_u \times n}$ and $D_c \in R^{n_u \times n_y}$.

Definition 3. LPV closed-loop system.

An LPV closed-loop system is defined by:

$$CL(\theta) : \begin{bmatrix} \dot{\eta} \\ z \end{bmatrix} = \begin{bmatrix} \mathscr{A}(\theta) & \mathscr{B}(\theta) \\ \mathscr{C}(\theta) & \mathscr{D}(\theta) \end{bmatrix} \begin{bmatrix} \eta \\ w \end{bmatrix} \tag{6}$$

where η, w and z are the state, the input and output, respectively, of the closed-loop system. $\theta(.) \in \Theta$ is the set of the varying parameters associated to the controller. $\mathscr{A} \in R^{n \times n}$, $\mathscr{B} \in R^{n \times n_w}$, $\mathscr{C} \in R^{n_z \times n}$ and $\mathscr{D} \in R^{n_w \times n_z}$.

The aim of the LTI/$\mathscr{H}_\infty$ synthesis is to minimize the $\mathscr{H}_\infty$ norm (γ_∞) of a system, while ensuring the internal stability ($K > 0$) (see Boyd et al., 1994). From the linear dissipative systems theory, finding such a controller leads to the Bounded Real Lemma (BRL) given by inequality (7) (see Apkarian & Gahinet, 1995; Chilali et al., 1999; Scherer et al., 1997).

$$\begin{bmatrix} \mathscr{A}^{\mathrm{T}}K + K\mathscr{A} & K\mathscr{B} & \mathscr{C}^{\mathrm{T}} \\ \mathscr{B}^{\mathrm{T}}K & -\gamma_\infty^2 I & \mathscr{D}^{\mathrm{T}} \\ \mathscr{C} & \mathscr{D} & -I \end{bmatrix} < 0 \tag{7}$$

Inequality (7) is a bilinear matrix inequality (BMI) so far, hence a non-convex problem has to be solved. Via a change of basis expressed in Scherer et al. (1997), and assuming $D_{22} = 0$ (in (4)), a non-conservative LMI (8) that expresses the same problem in a tractable way for semi-definite programs (SDP) can be found (with decision matrices X, Y, $\widetilde{A}$, $\widetilde{B}$, $\widetilde{C}$ and $\widetilde{D}$).

Solving (8) leads to the $\mathscr{H}_\infty$ optimal solution. Then, choosing M and N such that $MN^{\mathrm{T}} = I_n - XY$, the controller is obtained solving:

$$\begin{cases} \widetilde{D} = D_c \\ \widetilde{C} = D_c C_2 X + C_c M^{\mathrm{T}} \\ \widetilde{B} = Y B_2 D_c + N B_c \\ \widetilde{A} = YAX + YB_2 D_c C_2 X + N B_c C_2 X + Y B_2 C_c M^{\mathrm{T}} + N A_c M^{\mathrm{T}} \end{cases}$$

$$\begin{bmatrix} A\mathbf{X} + \mathbf{X}A^{\mathrm{T}} + B_2\widetilde{\mathbf{C}} + \widetilde{\mathbf{C}}^{\mathrm{T}}B_2^{\mathrm{T}} & * & * & * \\ \widetilde{\mathbf{A}} + A^{\mathrm{T}} + C_2^{\mathrm{T}}\widetilde{\mathbf{D}}^{\mathrm{T}}B_2^{\mathrm{T}} & \mathbf{Y}A + A^{\mathrm{T}}\mathbf{Y} + \widetilde{\mathbf{B}}C_2 + C_2^{\mathrm{T}}\widetilde{\mathbf{B}}^{\mathrm{T}} & * & * \\ B_1^{\mathrm{T}} + D_{21}^{\mathrm{T}}\widetilde{\mathbf{D}}^{\mathrm{T}}B_2^{\mathrm{T}} & B_1^{\mathrm{T}}\mathbf{Y} + D_{21}^{\mathrm{T}}\widetilde{\mathbf{B}}^{\mathrm{T}} & -\gamma_\infty I_{n_u} & * \\ C_1\mathbf{X} + D_{12}\widetilde{\mathbf{C}} & C_1 + D_{12}\widetilde{\mathbf{D}}C_2 & D_{11} + D_{12}\widetilde{\mathbf{D}}D_{21} & -\gamma_\infty I_{n_y} \end{bmatrix} < 0$$

$$\begin{bmatrix} \mathbf{X} & I_n \\ I_n & \mathbf{Y} \end{bmatrix} > 0 \tag{8}$$

In the LPV framework, the controller has to reach these objectives for the whole set of varying parameters. Hence the previous BRL (7) becomes,

$$\begin{bmatrix} \mathscr{A}(\theta)^{\mathrm{T}}K + K\mathscr{A}(\theta) & K\mathscr{B}(\theta) & \mathscr{C}(\theta)^{\mathrm{T}} \\ \mathscr{B}(\theta)^{\mathrm{T}}K & -\gamma_\infty^2 I & \mathscr{D}(\theta)^{\mathrm{T}} \\ \mathscr{C}(\theta) & \mathscr{D}(\theta) & -I \end{bmatrix} < 0 \tag{9}$$

that can be turned into an LMI in the same way as described in Eq. (8), where θ holds for the varying parameters. As θ is varying between upper ($\overline{\theta}$) and lower ($\underline{\theta}$) bounds, the LMI based problem results in an infinite set of LMIs to solve. Different approaches to reduce this problem into a finite number of LMIs are commonly used (see Biannic, 1996):

(1) gridding parameter space;
(2) linear fractional transformation (LFT);
(3) polytopic set of parameters.

In this paper the polytopic approach is used, which is appropriate when the parameter dependency enters in a linear way in the system definition, and when the number of varying parameters is small (see, e.g., Biannic, 1996; Zin et al., 2006). Applied to the $\mathscr{H}_\infty$ problem, such an approach consists in finding a common constant Lyapunov function $K > 0$ and a minimal γ_∞ that solve the previous LMI problem at each vertex of the polytope defined by system (4). Then the applied control is a convex combination of these controllers and can be expressed as follows (e.g. Biannic, 1996; Bruzelius, 2004; Zin, 2005):

$$S(\theta) = \sum_{k=1}^{2^i} \alpha_k(\theta) \begin{bmatrix} A_{c_k} & B_{c_k} \\ C_{c_k} & D_{c_k} \end{bmatrix} \tag{10}$$

where

$$\alpha_k(\theta) = \frac{\prod_{j=1}^{i} |\theta(j) - \mathscr{C}^c(\Theta_k)_j|}{\prod_{j=1}^{i} (\overline{\theta}(j) - \underline{\theta}(j))} \tag{11}$$

and

$$\sum_{k=1}^{2^i} \alpha_k(\theta) = 1, \quad \alpha_k(\theta) > 0 \tag{12}$$

where i is the number of varying parameters and $k = 2^i$, the number of vertices of the polytope. $\overline{\theta}$ and $\underline{\theta}$ are the upper and lower bounds of a parameter, respectively. Finally, $\mathscr{C}^c(\Theta_k)$ represents the complementary of Θ_k, which is simply the kth vertex of the polytope (Biannic, 1996; Zin, 2005).

6. Main result: LPV based robust semi-active suspension control design

The controller synthesis is based on the model described in (1) where $F_k(z_{def}) = k.z_{def}$ and $F_c(\dot{z}_{def}) = c.\dot{z}_{def}$ are linear functions (c and k are given in Table 1). The control law, applied on model (2), is then given by

$$u = -c(\dot{z}_{def}) + u^{\mathscr{H}_\infty} \tag{13}$$

where c is the nominal linearized damping coefficient of the considered semi-active damper (for the M-R damper, c nominal is the one given to provide nominal performances, e.g. no current is applied to the system) and $u^{\mathscr{H}_\infty}$ the additional force provided by the controller. To account for actuator limitations shown in Section 3, one develops a new method based on the LPV polytopic theory using the $\mathscr{H}_\infty$ controller synthesis approach.

6.1. Semi-active proposed approach and scheduling strategy

In order to meet the semi-activeness and performance requirements, the generalized control scheme is chosen as in Fig. 9.

The generalized block scheme incorporates the weighting functions (W_{z_r}, W_n, W_{z_s}, $W_{z_{us}}$ and $W_u(\rho)$), the static semi-active actuator model, $D(\dot{z}_{def})$, as presented in Section 3 (Figs. 6 and 7), i.e. the upper and lower saturations of the considered semi-active actuator that belong to the semi-active force/deflection speed space given in Fig. 4.

Then, $\rho(\varepsilon)$ is the scheduling parameter, function of $\varepsilon = u - v$, the difference between the computed force and the achievable one (a force is an achievable force if it is between the upper and lower bounds of the semi-active considered actuator, as illustrated by the static model $D(\dot{z}_{def})$ in Fig. 7), that will be used to satisfy the dissipative damper constraints (see next subsection). The idea is to add control when the required force is in the allowed quadrants, i.e. when $\varepsilon = 0$, and otherwise to rely on the passive law designed by the car manufacturer when the forces are outside the allowable space, i.e. when $\varepsilon \neq 0$. For that purpose the following scheduling strategy $\rho(\varepsilon)$ is introduced:

$$\rho(\varepsilon) = 10\frac{\mu\varepsilon^4}{\mu\varepsilon^4 + 1/\mu} \tag{14}$$

where μ modifies the slope of the $\rho(\varepsilon)$ function. In the proposed case, μ is chosen sufficiently high $(= 10^8)$ to ensure the semi-active control (see next subsection). In Fig. 10, the $\rho(\varepsilon)$ function is given different values of μ.

Remark.

- Note that the defined $\rho(\varepsilon)$ function is continuous and satisfies $\{\rho(\varepsilon) \in [0, 10]\}, \forall\varepsilon \in \mathbb{R}$. In other terms, ρ belongs to a convex set $[\underline{\rho}, \overline{\rho}]$. This point is essential in the LPV framework, as described in the previous section. For numerical implementation purposes, ρ will be considered as $[0.1, 10]$ instead of $[0, 10]$.
- Note $\varepsilon \neq 0$ $(\Leftrightarrow u \neq v)$ means that the required force is outside the allowed range. Conversely, $\varepsilon = 0 (\Leftrightarrow u = v)$ means that the force required by the controller is reachable for the considered semi-active actuator. This can be seen as a theoretical function of the usual "clipped" approach.

The idea is then to synthesize a controller $S(\rho)$, that is tuned according to this parameter in order to satisfy some performance objectives while ensuring the semi-activeness. In other words, to add force $(u^{\mathscr{H}_\infty})$ when it is possible and to rely on the passive solution when no force can be added. For that purpose the LPV synthesis framework is used with the use of the $\mathscr{H}_\infty$ tuning method.

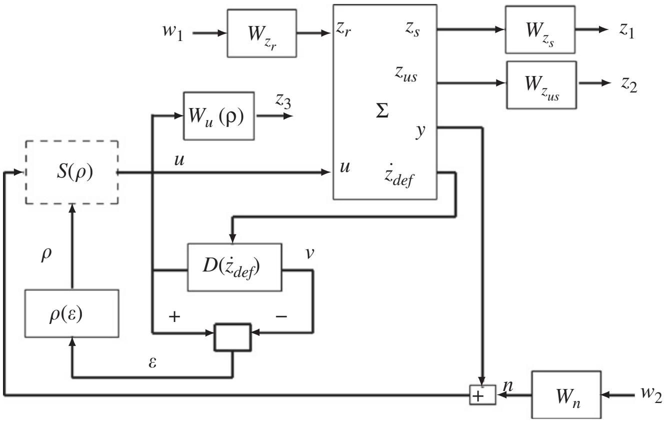


Fig. 9. General block diagram.

6.2. LPV design

As emphasized in the Introduction, the aim is to keep the control signal in the semi-active quadrants (Fig. 4). In other words, the control law aims at increasing/decreasing the damping coefficient. Here, in the LPV design presented in Fig. 9, $W_u(\rho)$, which is the performance criterion on the control signal, is ρ-dependent. Let us remember that in the $\mathscr{H}_\infty$ framework, this weight indicates how large the gain on the control signal can be. Choosing a high $W_u(\rho) = \rho$ forces the control signal to be low, and conversely. Hence, when ρ is large, the control signal is so penalized that it is practically zero, and the closed-loop behavior is the same as the passive quarter vehicle model one. Conversely, when ρ is small, the control signal is no more penalized; hence the controller can achieve performances in the allowed force/deflection speed space.

Let us consider the generalized plant description,

$$\begin{bmatrix} \dot{x} \\ z_\infty \\ y \end{bmatrix} = \begin{bmatrix} A(\rho) & B_\infty(\rho) & B \\ C_\infty(\rho) & D_{\infty w}(\rho) & D_{\infty u} \\ C & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ w_\infty \\ u \end{bmatrix} \tag{15}$$

where

$$\begin{cases} x = \begin{bmatrix} x_{quarter} & x_{weights} \end{bmatrix}^{\mathrm{T}} \\ z_\infty = \begin{bmatrix} W_{z_s} z_s, & W_{z_{us}} z_{us}, & W_u(\rho) u \end{bmatrix}^{\mathrm{T}} \\ w_\infty = \begin{bmatrix} W_{z_r}^{-1} z_r, & W_n^{-1} n \end{bmatrix}^{\mathrm{T}} \\ y = z_{def} \\ \rho \in \begin{bmatrix} \underline{\rho} & \overline{\rho} \end{bmatrix} \end{cases}$$

where x is the concatenation of the linearized quarter vehicle model (obtained thanks to Eq. (1)) and the weighting function state variables, z_∞ the performance signals, w_∞ the weighted input, y the measured signal and ρ the varying parameter.

As introduced in the performance objectives described in Section 4, W_{z_s} (resp. $W_{z_{us}}$) is shaped in order to reduce the amplification of the suspended mass z_s (resp. z_{us}) between 0 and 5 Hz (resp. 0–20 Hz), W_{z_r} and W_n model ground disturbances (z_r) and measurement noise (n), respectively, and $W_u(\rho)$ is used to limit the control signal and achieve the semi-active constraint

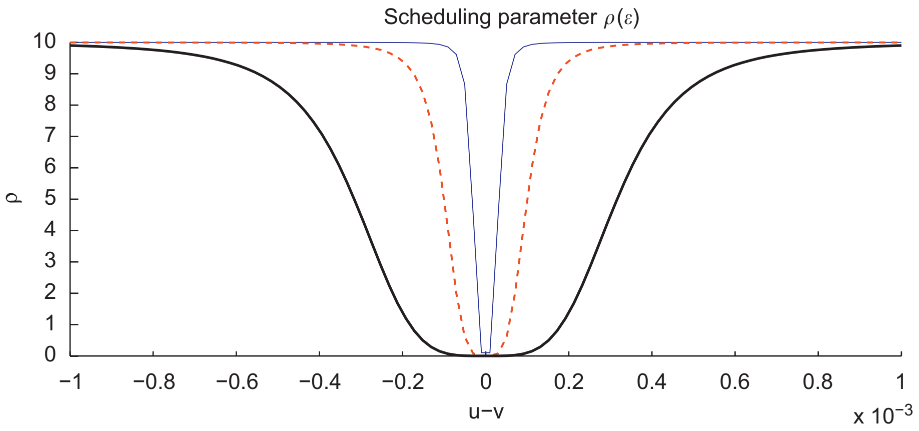


Fig. 10. $\rho(\varepsilon)$ solid thick (for $\mu = 10^6$), dashed (for $\mu = 10^7$) and solid thin (for $\mu = 10^8$).

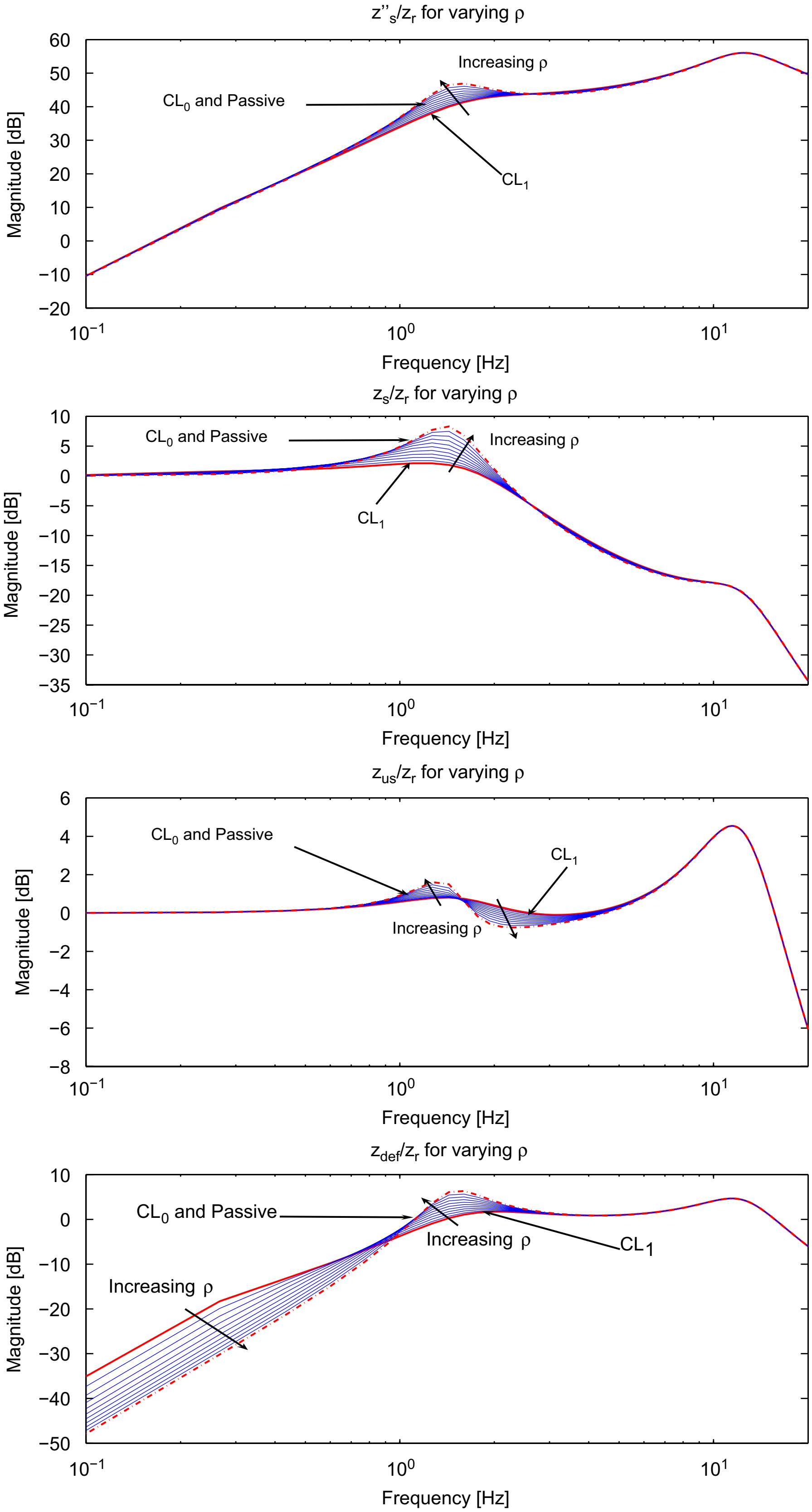


Fig. 11. Bode diagram of the closed-loop for different values of $\rho \in [0.1; 10]$ (from top to bottom) $\ddot{z}_s/z_r$, z_s/z_r, z_{us}/z_r, z_{def}/z_r.

(see next subsection). The weight functions are given by

$$\begin{cases} W_{z_s} = 2\dfrac{2\pi f_1}{(s+2\pi f_1)} \\ W_{z_{us}} = 1\dfrac{2\pi f_2}{(s+2\pi f_2)} \\ W_{z_r} = 7\times 10^{-2} \\ W_n = 10^{-4} \\ W_u(\rho) = \rho \in [0.1 \;\; 10] \end{cases}$$

where $f_1 = 3\,\text{Hz}$ and $f_2 = 5\,\text{Hz}$. Applying the LMI based polytopic LPV/$\mathscr{H}_\infty$ control synthesis (described through LMIs (8)) to the generalized plant (15) leads to two controllers $S(\underline{\rho})$ and $S(\overline{\rho})$, hence to two closed-loops ($CL(\underline{\rho})$ and $CL(\overline{\rho})$). Then the applied control law is the convex combination of these two controllers, function of ρ, as described in the following equation:

$$u^{\mathscr{H}_\infty} = \left(\frac{|\rho-\overline{\rho}|}{\overline{\rho}-\underline{\rho}}S(\underline{\rho}) + \frac{|\rho-\underline{\rho}|}{\overline{\rho}-\underline{\rho}}S(\overline{\rho})\right)y \qquad (16)$$

Hence, the controller $S(\rho)$ and the closed-loop $CL(\rho)$ can be expressed as the following convex hull:

$$S(\rho) \in co\{S(\underline{\rho}), S(\overline{\rho})\}$$

$$CL(\rho) \in co\{CL(\underline{\rho}), CL(\overline{\rho})\}$$

or simply (in Fig. 11) by,

$$S(\rho) \in co\{S_1, S_0\}$$

$$CL(\rho) \in co\{CL_1, CL_0\}$$

The system closed-loop frequency responses obtained with the LPV controllers (for different values of $\rho \in [0.1; 10]$) are given in Fig. 11.

Note that these closed-loop transfers provide a good frequency behavior (good performances) when ρ is small (CL_1), improving comfort in the range 0–5 Hz and road holding in the range 0–20 Hz, while the other one perfectly matches with the passive suspension (CL_0) which for the considered Renault Mégane car is road holding oriented.

Note also that a major interest in using the LPV design is that the internal stability of the closed-loop system is ensured for all $\rho \in [\underline{\rho}; \overline{\rho}]$, hence the system is kept under control.

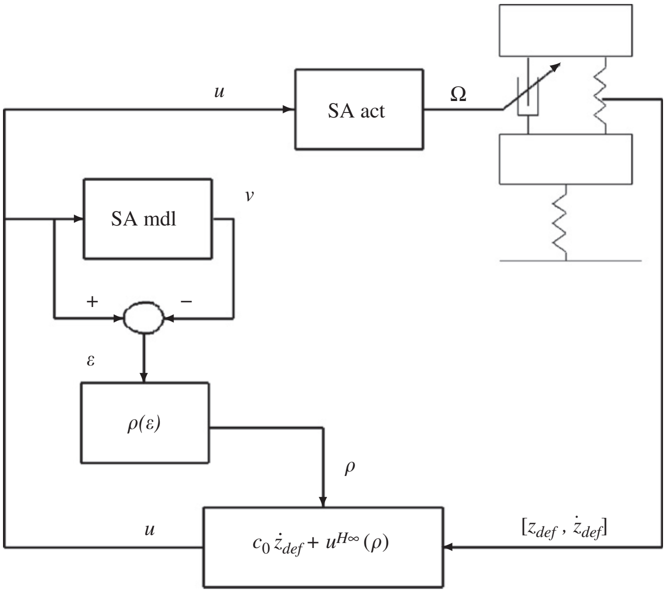
Remark. The Bode diagrams given in Fig. 11 show the system frequency performance according to varying ρ. Remember that the ρ parameter is not chosen by the driver, but imposed by the semi-active model of the damper.

6.3. Controller reduction

One of the main drawbacks of the $\mathscr{H}_\infty$ design is that the designed controller is of the same order as the generalized plant. In this case, the system is of order 4 and two dynamic weighting functions are introduced, i.e. two states; hence, the two obtained controllers are of order 6 which may be a problem for implementation issues. Here the controller order reduction is proposed to make easier the implementation on a real application.

Hankel singular values define the energy of each state in the system. Keeping larger energy states of a system preserves most of its characteristics in terms of stability, frequency and time responses (see Glover, 1984). Model reduction techniques, based on the Hankel singular values, can achieve a reduced-order model and preserve the majority of the system characteristics. Let us define the Hankel singular values as

$$\sigma_H = \sqrt{\lambda_i(PQ)}$$



u
SA act
Ω
SA mdl
v
+
−
ε
ρ(ε)
ρ
u
$c_0\,\dot{z}_{def} + u^{H\infty}(\rho)$
$[z_{def}, \dot{z}_{def}]$

Fig. 12. Implementation sketch diagram.

where P and Q are controllability and observability Grammians, and λ_i the eigenvalue function. One can perform such a Hankel measure on the two obtained controllers (which are both stable). The reduced controller obtained (of order 4) is verified against the initial performance criteria (used in the original design). This can be done by plugging the system matrices of the reduced controller in the LMIs and solving the corresponding analysis problem. Comparison of the obtained performances between full and reduced controller will be given in the next section.

6.4. Implementation issues

Here, a sketch of implementation in order to apply the proposed strategy is given. Fig. 12 shows the implementation synopsis, for any kind of semi-active actuators.

“SA mdl” is the static model of the semi-active damper considered in the application (here $D(\dot{z}_{def})$, presented in Section 3), $\rho(\varepsilon)$ is the scheduling law given in (14). “SA act” is the considered actuator and its inner loop (e.g. inverse model) of the semi-active damper that modifies the damping coefficient. Then, u, v, ε and ρ are the control and scheduling signals as described before. Finally, Ω is used here to specify the set of input parameters of the real controllable damper “SA act”. As an illustration, for an M-R damper, the real control input is I_{MRD}, the current that modifies the magneto-rheological fluid viscosity. Hence, in the case of the M-R damper actuator, the force u has to be converted into a current (by the means of tables or electrical laws for instance), hence $\Omega = I_{MRD}$. For other kinds of semi-active actuators, control input can be some mechanical, pneumatic elements or other damper input variables depending on the chosen technology (see Delphi, 2007; Lord, 2007; Sachs, 2007).

Remark.

- This scheme should be the same for any suspension control approaches. The advantage here is that the required force u always remains in the allowed force range of the considered controlled semi-active actuator (thanks to the proposed LPV approach).
- Note also that if a dynamical model of the considered actuator (gain, bandwidth limit, etc.) is available, it may be included in the synthesis by the mean of the $W_u(\rho)$ weighting function.

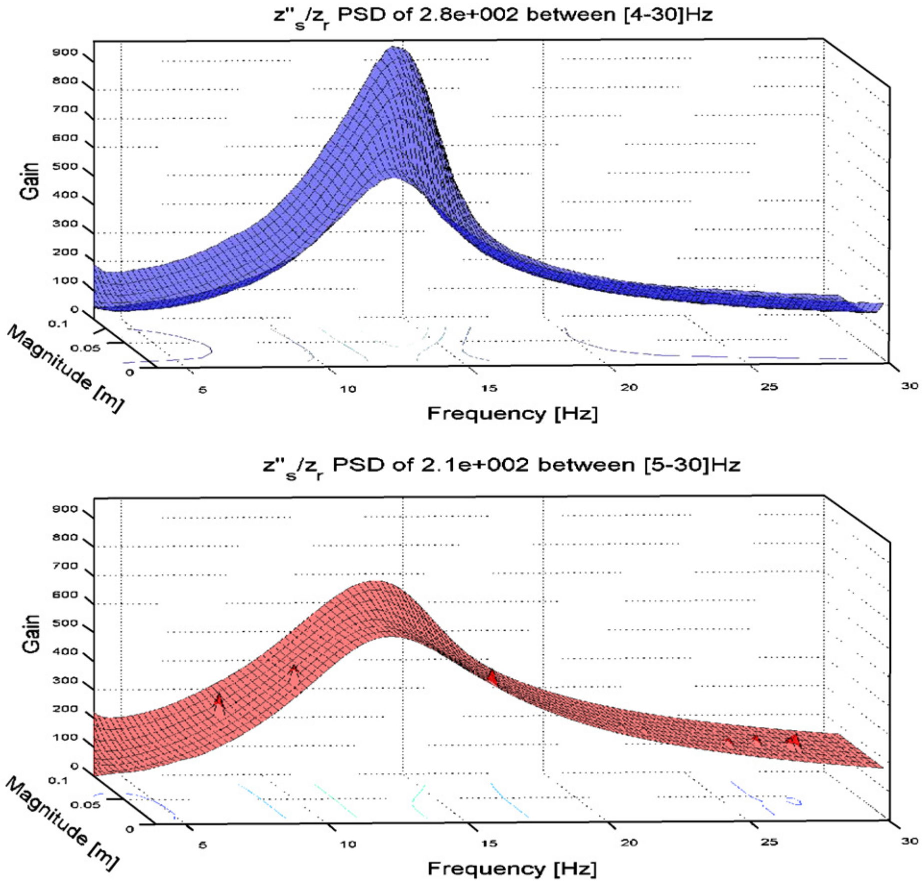


Fig. 13. Frequency response of $\ddot{z}_s/z_r$ for the passive (top) and controlled (bottom) quarter vehicle.

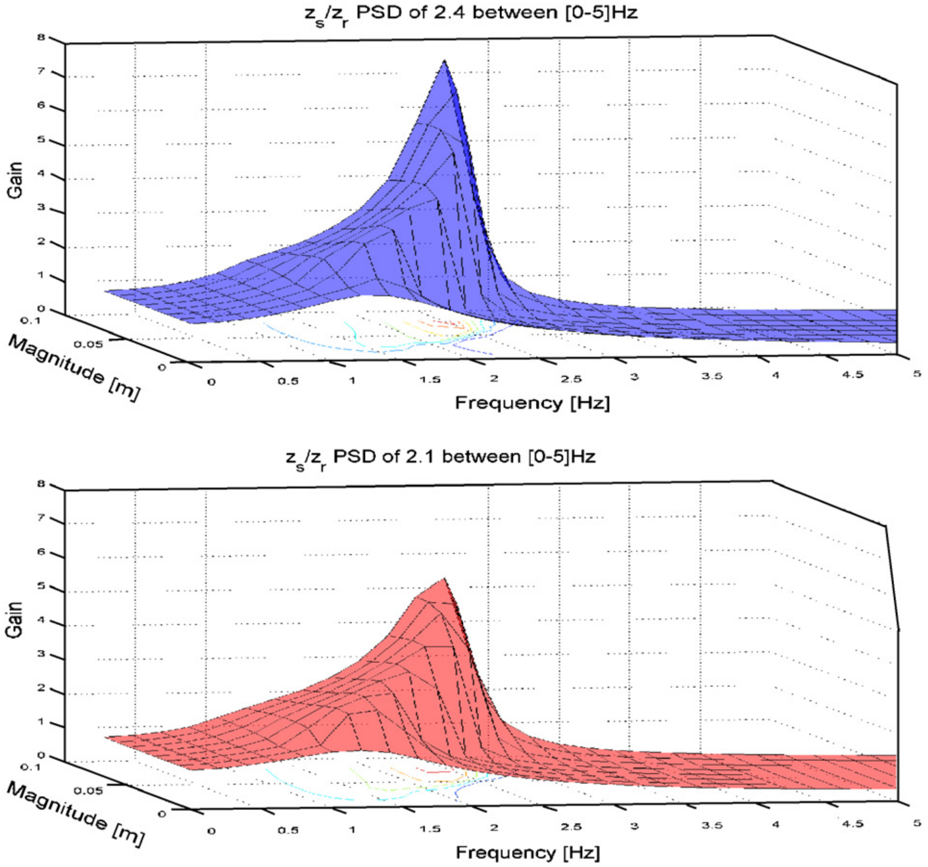


Fig. 14. Frequency response of z_s/z_r for the passive (top) and controlled (bottom) quarter vehicle.

7. Time and frequency results

In this section a validation of the proposed control approach is performed. First, performance evaluation, based on the previously introduced criterion, is done to show the control improvement. Then time domain simulations are performed to check that the semi-active constraint is fulfilled.

7.1. *Frequency results and criteria evaluation*

The performance criterion (3) is used to evaluate both passive and controlled (using the presented M-R damper as the semi-active actuator) nonlinear quarter car models. Let us recall that the parameters used in the simulations for the passive reference model correspond to the “Renault Mégane Coupé” one, a car satisfying a good road holding and handling as well behavior. This means that the proposed semi-active control will not improve the road holding characteristics significantly.

Figs. 13–16 show the nonlinear frequency response and the PSD value of $\ddot{z}_s/z_r$, z_s/z_r, z_{us}/z_r and z_{def}/z_r, respectively, for frequency and magnitude range of interest (as described previously), of the passive and controlled quarter car vehicle.

Note that the results are plotted using linear scales for the gain and the frequency. These plots allow to check easily that the proposed control design improves the quarter car behavior in the frequency region of interest. Moreover, in Table 2 the improvement is evaluated as (passive PSD − controlled PSD)/passive PSD.

The strategy leads to great improvements concerning comfort objectives (25% and 12%) and moderate ones concerning road holding (7% and 8%) which is very satisfactory for the “Renault Mégane Coupé”. The same evaluation is computed for the reduced order LPV controller (Table 3).

Results are nearly the same for the full and reduced order controller which is consistent according to the Hankel singular values computed. Note that this can be interesting and very useful for implementation issues.

7.2. *Time simulation results*

Finally, to validate in a more practical way the performances of the proposed approach and to check that the forces provided by the controller are semi-active, time simulations on the nonlinear quarter car model for different situations and analysis of the car behavior and the force/deflection speed space diagram are performed. Here, all simulations are done using the reduced LPV controller (order 4).

7.2.1. *Step road disturbance*

First a step road disturbance of magnitude −3 cm is generated at time 1 s. The suspension control force and the chassis and wheel behaviors are shown in Figs. 17 and 18.

Fig. 17 clearly shows that the forces provided by the controller remain in the semi-active allowed space and in the achievable range of the considered Delphi M-R damper. Then Fig. 17 shows how the ρ parameter is varying according to the step. Notice that between 1 and 1.08 s, $\rho = \underline{\rho}$, which means that controller output is in the allowed semi-active quadrant, then ρ raises to $\overline{\rho}$, which means that the required force is outside the allowed area. This plot shows how the scheduling is performed.

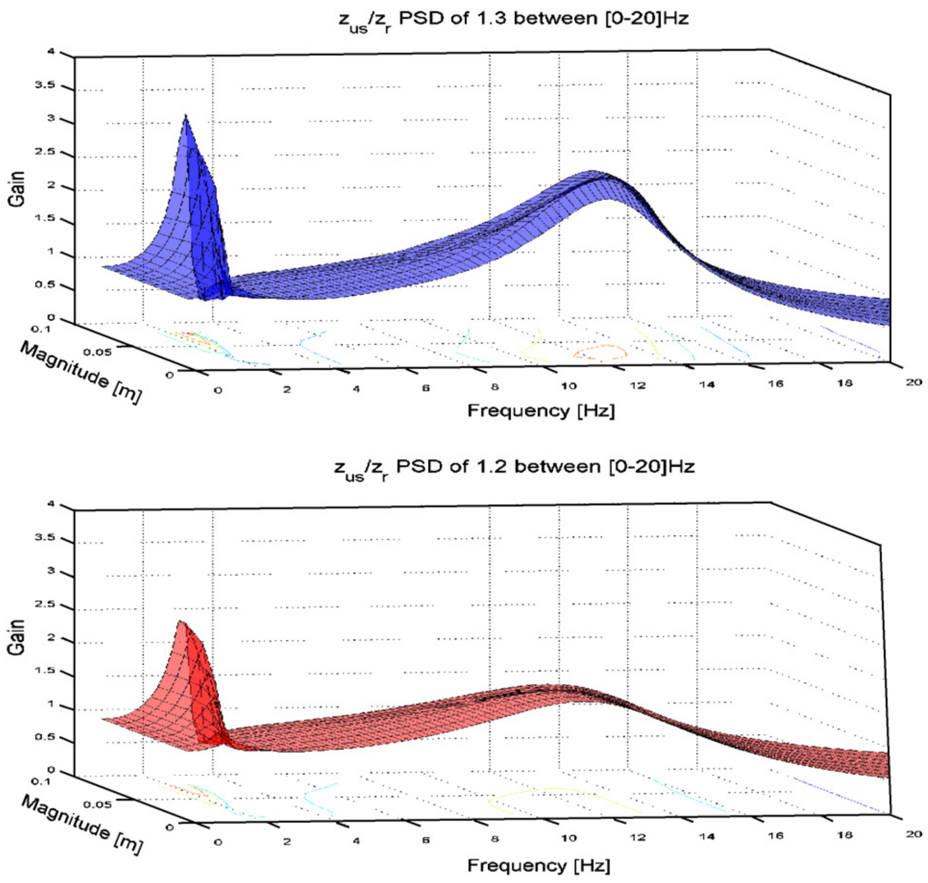


Fig. 15. Frequency response of z_{us}/z_r for the passive (top) and controlled (bottom) quarter vehicle.

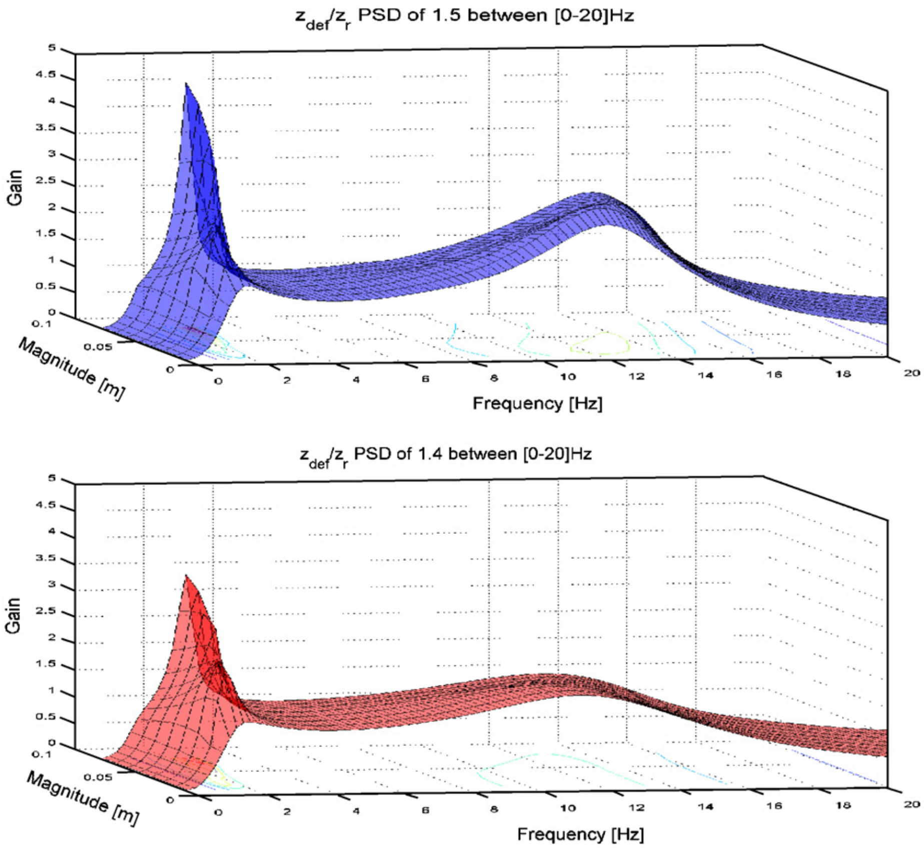


Fig. 16. Frequency response of z_{def}/z_r for the passive (top) and controlled (bottom) quarter vehicle.

Table 2
Passive vs. full order (order 6) controlled PSD

Signal	Passive PSD	Controlled PSD	Improvement (%)
$\ddot{z}_s/z_r$	277.1	206.7	25.402
z_s/z_r	2.402	2.110	12.142
z_{us}/z_r	1.299	1.208	7.0038
z_{def}/z_r	1.531	1.408	8.0212

Table 3
Passive vs. reduced order (order 4) controlled PSD

Signal	Passive PSD	Controlled PSD	Improvement (%)
$\ddot{z}_s/z_r$	277.1	206.8	25.350
z_s/z_r	2.402	2.110	12.142
z_{us}/z_r	1.299	1.208	7.0047
z_{def}/z_r	1.531	1.408	8.0191

Fig. 18 shows that the proposed controller attenuates undesired oscillations for both the chassis and the wheel. These results confirm the one obtained thanks to the frequency plots.

7.2.2. *Random road disturbance*

Here a random road profile is applied during a long duration (20 s) with magnitude between +1 cm and −1 cm.

As previously observed, results match the semi-active constraint (Fig. 19) and keep both high comfort and road holding performances (Fig. 20).

8. Conclusion and future works

In this article, a new strategy to ensure the dissipative constraint for a semi-active suspension is introduced, while keeping the advantages of the $\mathscr{H}_\infty$ control design. Interests of such an approach compared to existing ones are:

1. Flexible design: possibility to apply $\mathscr{H}_\infty$, $\mathscr{H}_2$, pole placement, mixed criteria, etc.
2. Measurement: only the suspension deflection sensor (and its first derivative) is required.
3. Computation: synthesis leads to two LTI controllers (of order 4) and a simple scheduling strategy based on a static actuator model (no on-line optimization process involved).
4. Internal stability is preserved and the obtained performances ensure that the controlled semi-active suspension always improve the ones of the passive suspension.
5. Implementation: solution tractable for any kind of semi-active actuators.

The proposed control design shows good performances, through both frequency based industrial criteria and time simulation experiments performed on a nonlinear model. Hence this new semi-active strategy exhibits significant improvements on the achieved performances. Moreover, compared to two-state Skyhook, LQ and MPC techniques, the implementation of such a controller results in a low cost solution in terms of controller order (time consumption) and sensor requirements which is one of the key points in all embedded solutions. The proposed approach although having been illustrated by the use of M-R dampers, can be applied to any semi-active actuators.

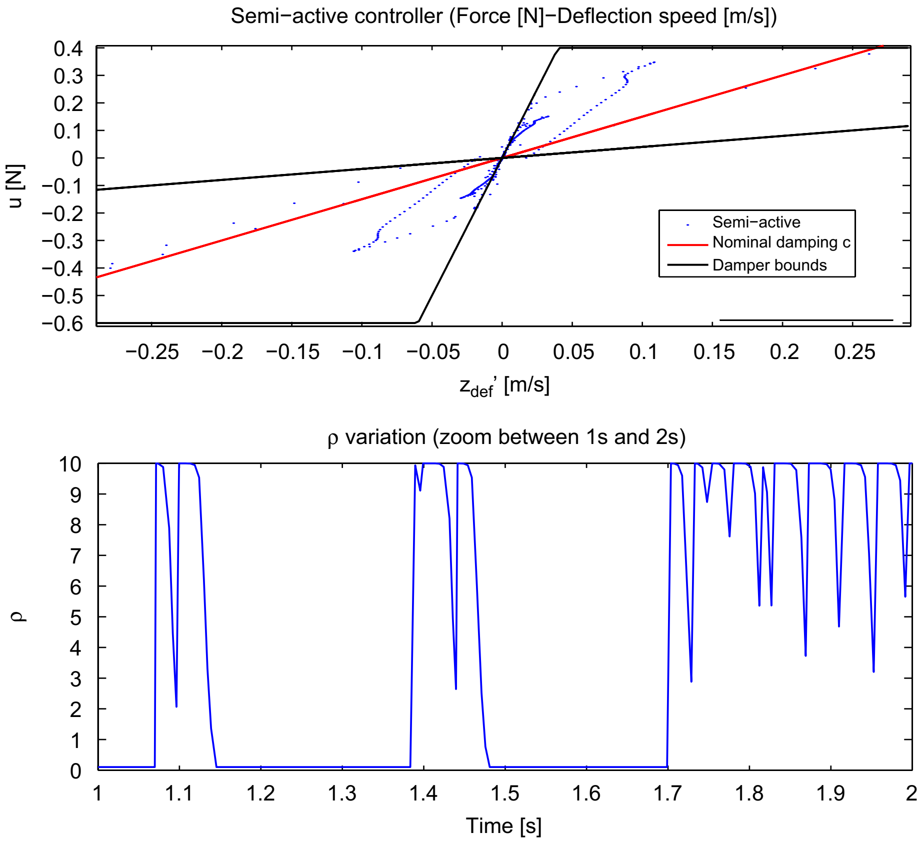


Fig. 17. Force/deflection speed diagram in response to a step road disturbance. ρ variation between 1 and 2 s.

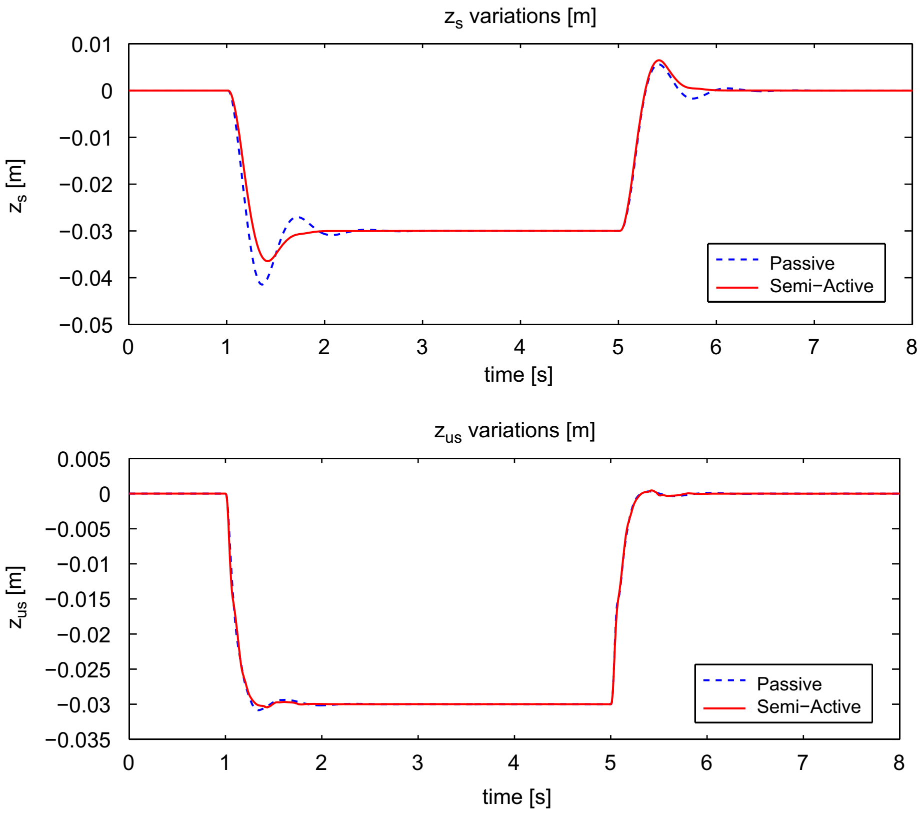


Fig. 18. Chassis response, z_s (top) and wheel z_{us} (bottom) in response to a step road disturbance.

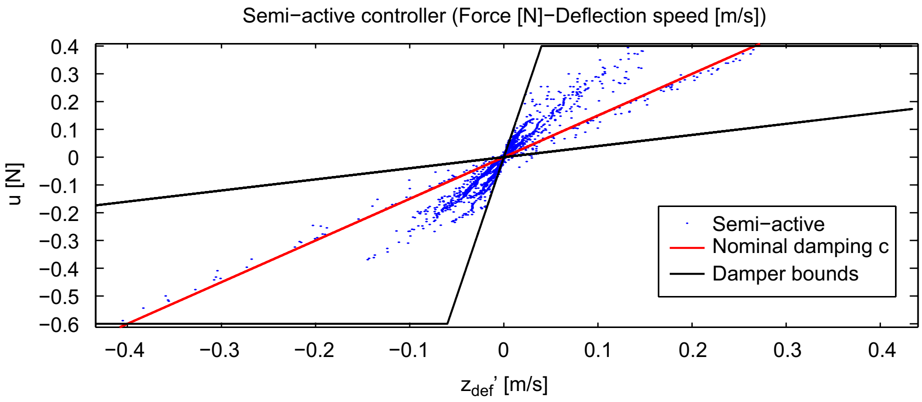


Fig. 19. Force/deflection speed diagram in response to a random road profile.

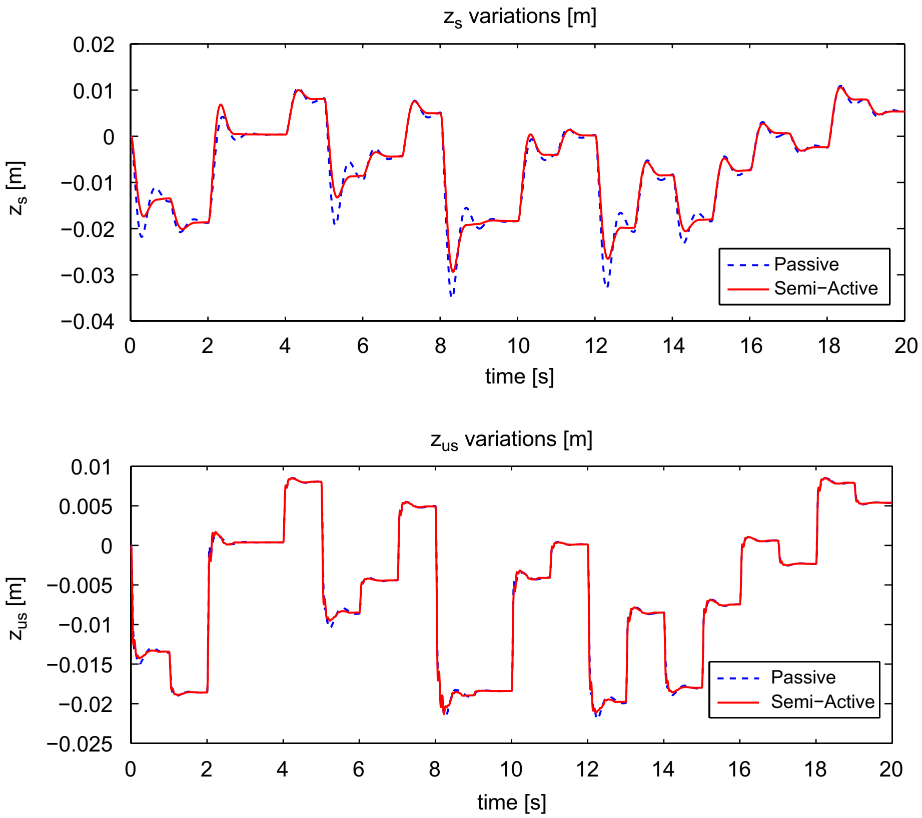


Fig. 20. Chassis response, z_s (top) and wheel z_{us} (bottom) in response to a random road profile.

In future works discretization and implementation issues of such a controller taking into account the sampling time, the quantization problems, etc., will be investigated. As long as this controller is built in the robust framework, robustness analysis could be an interesting extension of this work. Finally, implementation on the suspension test bench available at the Tecnologico of Monterrey could be an interesting issue.

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